Fuzzy (and other) Logics.*

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1 Fuzzy Logic

A related topic, which we could have covered in this course and which may be of interest to linguists (especially syntax/semantics people) is Fuzzy Logic. Fuzzy logic is an alternative to probabilities for expressing concepts that are not completely definite.

Fuzzy logic deals with things that can be partial members of several sets at once. This is not the same idea that underlies probability: when we use a probabilistic description, we assume that an animal we see out of the corner of our eye is either a cat or a dog, but the night is dark and we don't know which.

An example where fuzzy logic might make sense is politics. A person does *not* have to be exclusively either a Tory or Labour. It does not make sense to say that "He is 62% likely to be Tory," because that statement carries the assumption that he is really either Tory or not, either Margaret Thatcher or Edward Heath.

The fuzzy description is subtly different: "She is 62% Tory," assumes that she is partially a Tory. She is also, simultaneously, a partial member of other political factions, including Labour, Communists, Libertarians, Greens and so forth. She has some options that the Tories would be happy to endorse, but she disagrees with them on other points.

As with probabilities, there are arithmetic operations on these fuzzy values that mimic all the normal rules of logic. The arithmetic is different from the arithmetic on probabilities, however. As with probabilities, fuzzy logic becomes identical to "normal" Boolean logic if all the partial memberships are either 0% or 100%, as one would hope. For instance, if a person is 100% Tory, he is 0% not-Tory.

However, when memberships are only partial, interesting things can happen. For instance, the law of the excluded middle no longer applies. Getting back to our example of someone who is 62% Tory, we find that she is (unsurprisingly) 38% not-Tory. Perhaps surprisingly, though, we find that she is 38% a member in the set of people who are both Tories and not-Tories. This is quite different from the logic of probabilities, where an animal has a zero percent chance of being both a cat and a non-cat[1].

Fuzzy logic suffers, though, because there is no obvious way to measure these fractional memberships. Unlike probability theory, where one can count the number of dogs in the room, count the number of total animals, and take the ratio to get an estimate for $P(dog)$, there is no trivial, unarguable way to measure how much of a Tory someone is. Consequently, despite much hoopla in the 1970s-1990s, fuzzy logic has had little impact. Fuzzy logic suffers because doing well-defined logic on ill-defined values is only partially satisfactory.

For further information on fuzzy logic, see [Kickert, 1978, web, 2004, Hajek, 2002, Ross, 1995].

As a further aside, a related logic is August Stern's "Matrix Logic" [Stern, 1988], which expands George Boole's True/False to a set of four values: True, False, both True and False, and neither True nor False. At a quick glance, it has some of the same ideas and features as fuzzy logic, though in a 4-valued discrete setting, rather than a continuous spectrum of values.

References

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[Kickert, 1978] Kickert, W. J. M. (1978). *Fuzzy Theories and Decision Making*. Martinus Nijhoff Social Sciences Division, Leiden, Boston, London. Distributed by Kluwer (Boston). ISBN 90-207-0760-4. Note that there is a chapter on "Fuzzy Linguistic Models.".

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[1] Neglecting genetic engineering...