This is applied philosophy: it all comes from the question “what is a galaxy?”

The Hotspot Decomposition

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Defining a galaxy in terms of a mass density is the right definition, but unfortunately, the density isn't easily observable.

What is a galaxy?

- A local minimum in the gravitational potential.

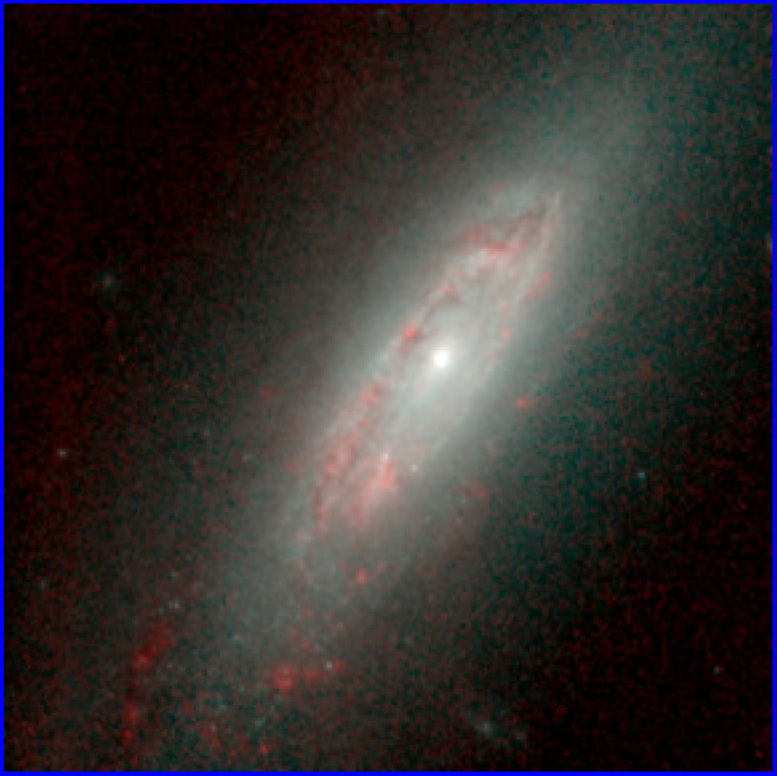
$$\nabla^2 \phi = \rho G$$

- A lot of dark matter
  - ϕ is very smooth, ρ is smooth.
- A sprinkling of 10^9 - 10^{14} stars, some gas, and some dust.
  - The density of stars is fairly smooth
  - The light isn't, because $L \propto M^4$

A massive star can be 10^8 times brighter than a faint one, and 10^4 time brighter than a typical star.

What does a typical galaxy look like?

Notice, below, the pronounced spiral structure. It would take a very complex equation to accurately represent this galaxy.



How do you model a galaxy?

- Parametrically
  - Parameters have physical meaning, can be connected to other models.
  - Pretty crude representation unless many terms involved.

$$I = I_0 e^{-r/r_0}$$

Surface Brightness

$$I = I_0 e^{-(r/r_0)^{1/4}}$$

Distance from center, with some elipticity.

- Nonparametrically
  - Ability to precisely represent the galaxy

What kind of nonparametric fit?

- Monotonically decreasing from a center $\frac{dI}{dr} < 0$ √

Algorithm enforces this. This is rigorously true for mass density, close to true for I.

- Approximate 2-fold symmetry √/2

This happens even without enforcement.

- Asymptotically approaching a constant

$$\frac{d^2 I}{dr^2} > 0, \quad \frac{dI}{dr} \to 0$$

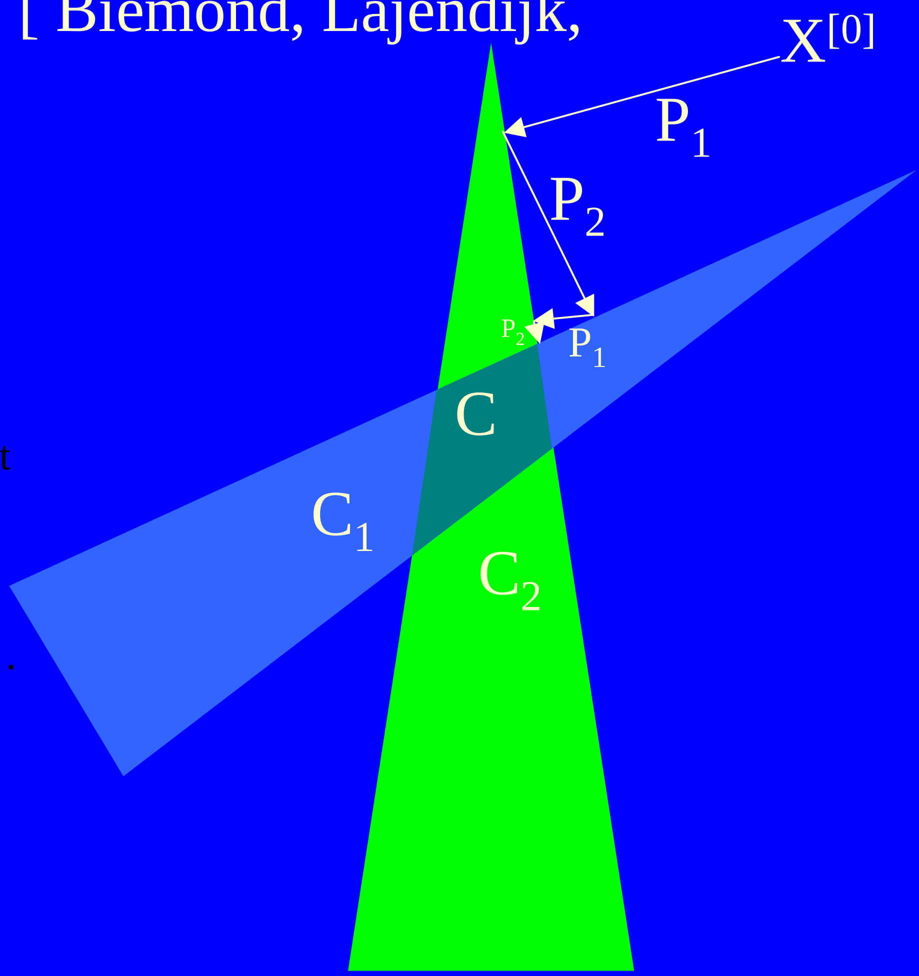
Mostly true, but not always (especially near center).

How do you implement a monotonic fit?

Use spatially local projection operators and the Method of Projections onto Convex Sets [Biemond, Lajendijk, Mersereau, 1990]

If C is a convex region which can be written as the intersection of a set of convex regions : $C = C_1 \cap C_2 \cap \cdots$, and if we have a set of projection operators P_i that project into the nearest point in C_i ,

then the sequence $x^{[i+1]} = P_1 P_2 \ldots x^{[i]}$ will converge to the nearest point in C .



Implementing the constraints

Consider a one-dimensional monotonicity constraint to enforce $x_1 \geq x_2$:

$$P(x_1, x_2) = \begin{cases} (x_1, x_2) \rightarrow (x_1, x_2), \text{ if } x_1 \geq x_2 \\ (x_1, x_2) \rightarrow (\bar{x}, \bar{x}), \text{ where } \bar{x} = (x_1 + x_2)/2, \text{ otherwise} \end{cases}$$

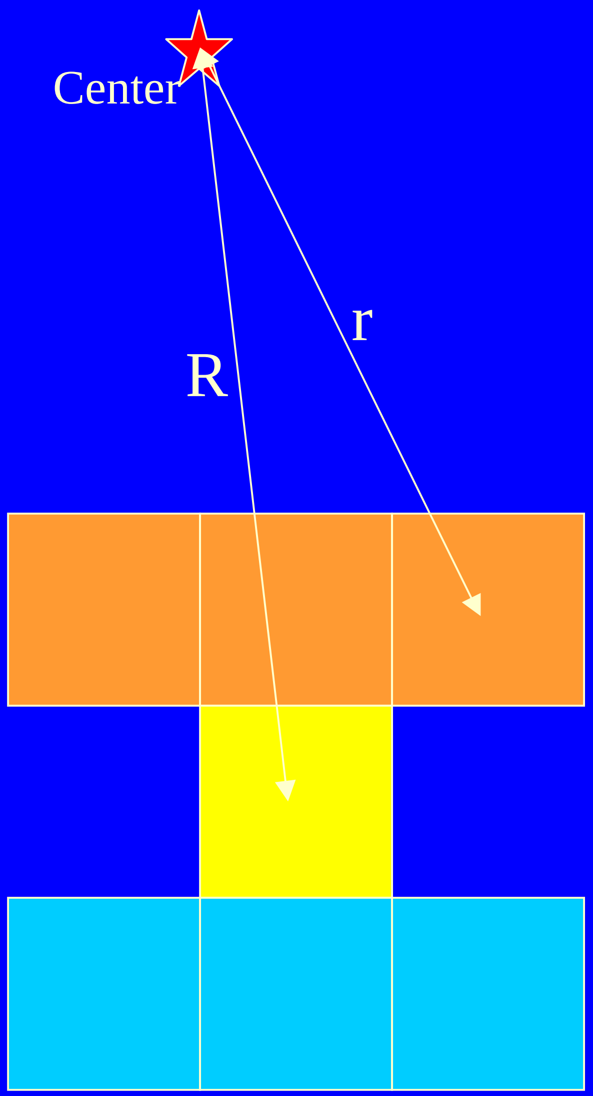
* P projects to the nearest point in the constraint region.

* The constraint region is convex:

Given any two pairs $a = (a_1, a_2)$ and $b = (b_1, b_2)$ that satisfy the constraint, then all points in between (i.e., $\alpha \cdot a + (1\text{-}\alpha) \cdot b$, *for* $0 \leq \alpha \leq 1$) do also.

So, we can use this P, repeatedly applied on each pixel, to make a one-dimensional image monotonic.

Building 2-dimensional constraints from 1-D constraints.

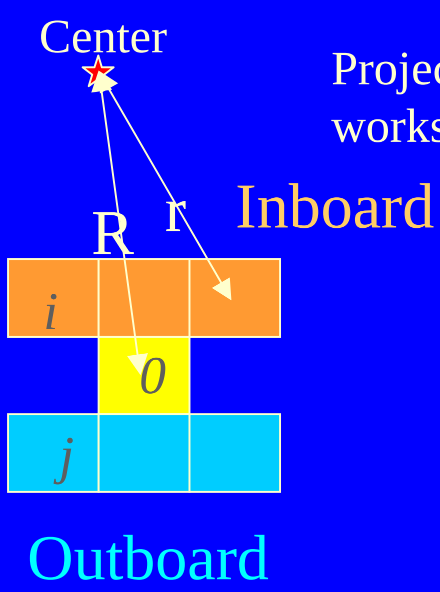


Each constraint and projection operator works on a small cluster of pixels.

Inboard neighbors:
any neighbors with $r < R-0.5$

Outboard neighbors:
any neighbors with $r > R+0.5$

Building 2-dimensional constraints from 1-D constraints.



Projection operator
works on a small cluster of pixels.

The cluster is defined to be monotonic iff :

$\exists$ an inboard neighbor, i, such that $I_i \geq I_0$, and

$\exists$ an outboard neighbor, j, such that $I_j \leq I_0$

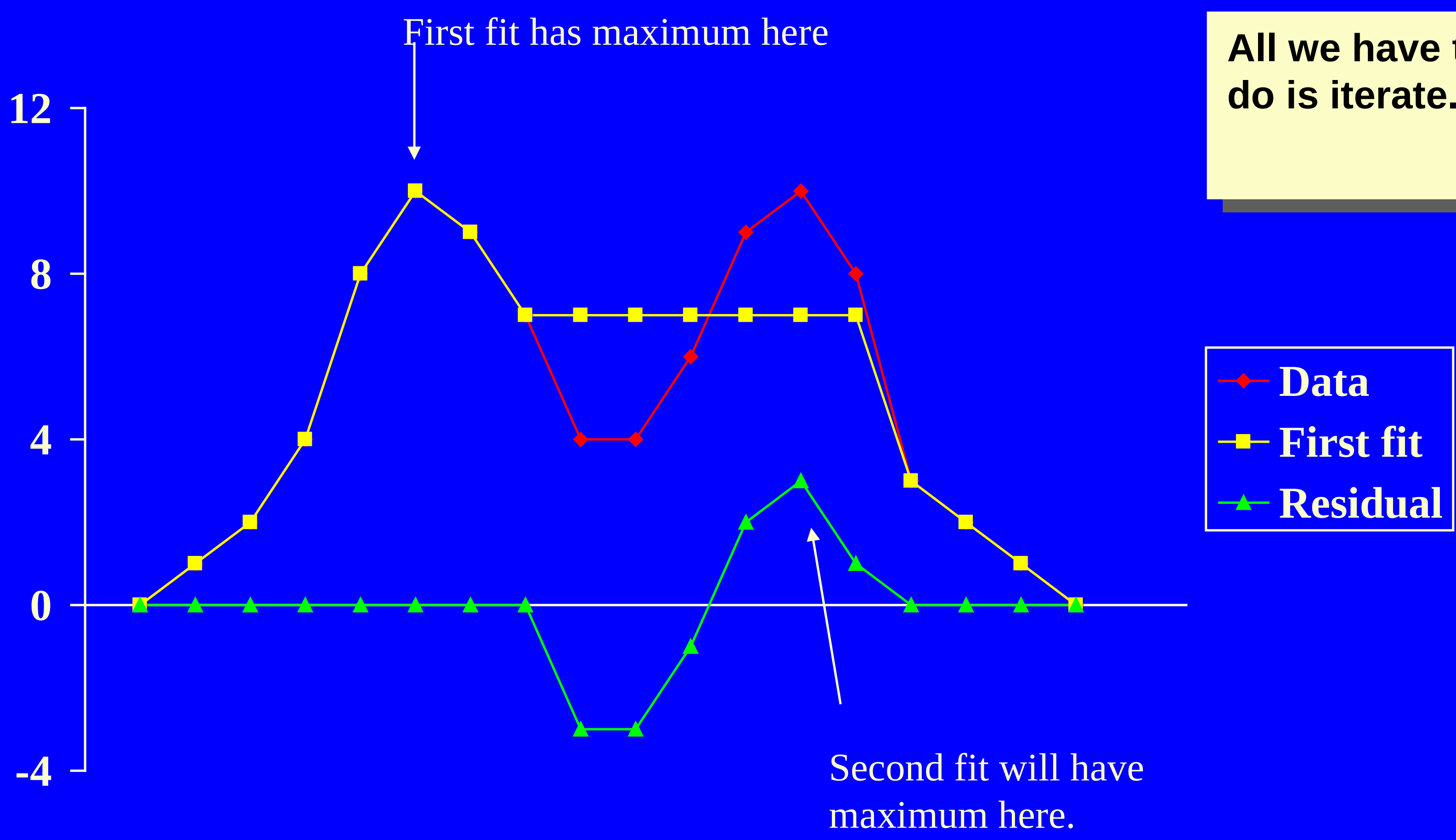
This implies monotonicity along a directed acyclic graph that starts at the center. Each node is further out than its predecessor.

If you didn't have the dual requirement that both outboard and inboard neighbors need to be constrained, you'd get a much rougher output image.

Choice is partially empirical (i.e. fits galaxies better), partially to make the local texture agree with one's intuitive definition of 'monotonic'.

You'd get lots of leaf nodes without the symmetrical constraint. High walls at the end of blind alleys.

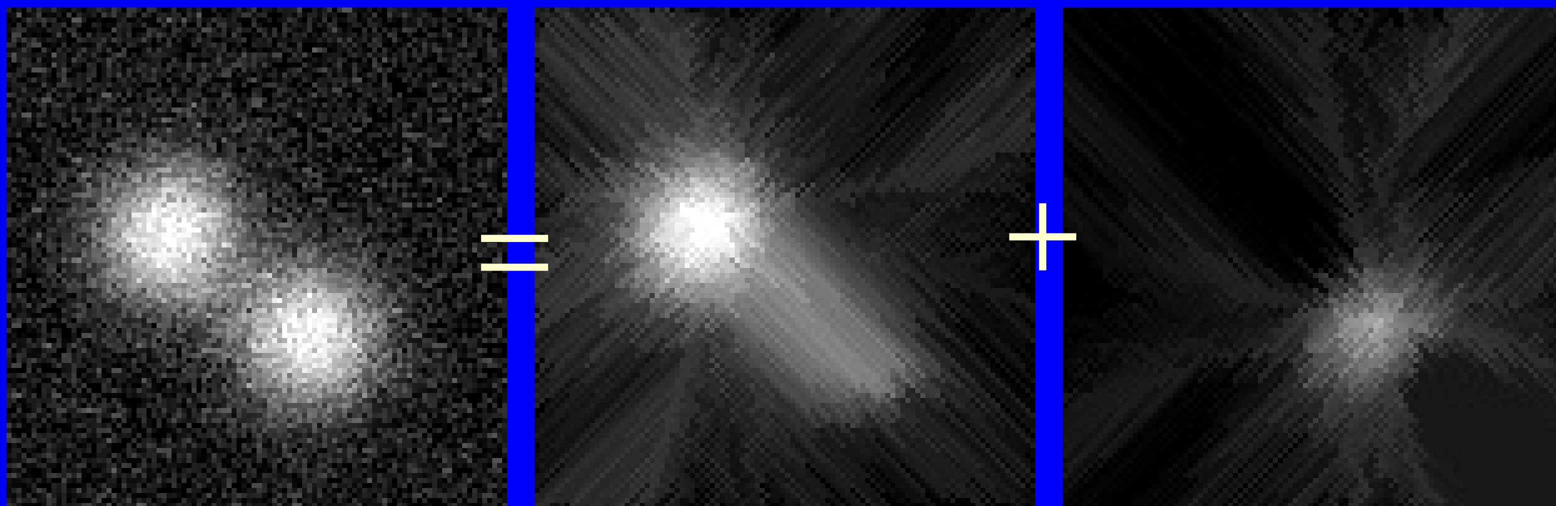
OK, so what about two galaxies?



All we have to do is iterate...

Asymptotic result: stable after about 3 iterations.

Two galaxies in two dimensions.

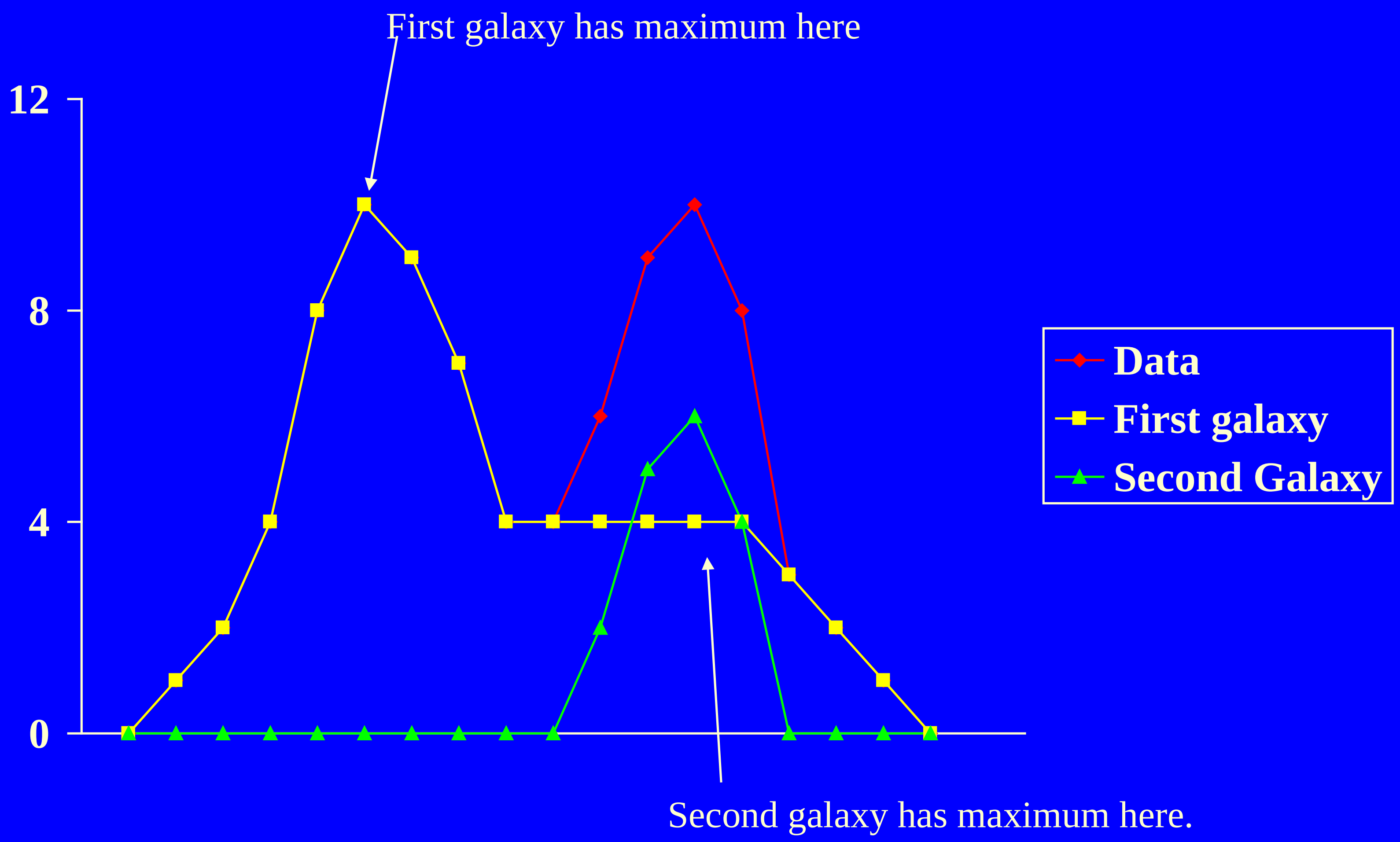


* Simple iteration converges very fast, but gets stuck on an asymmetric, order-dependent solution.

* The light in between the galaxies can just as well be assigned to either one, or both.

Residual

How does the iteration get trapped?



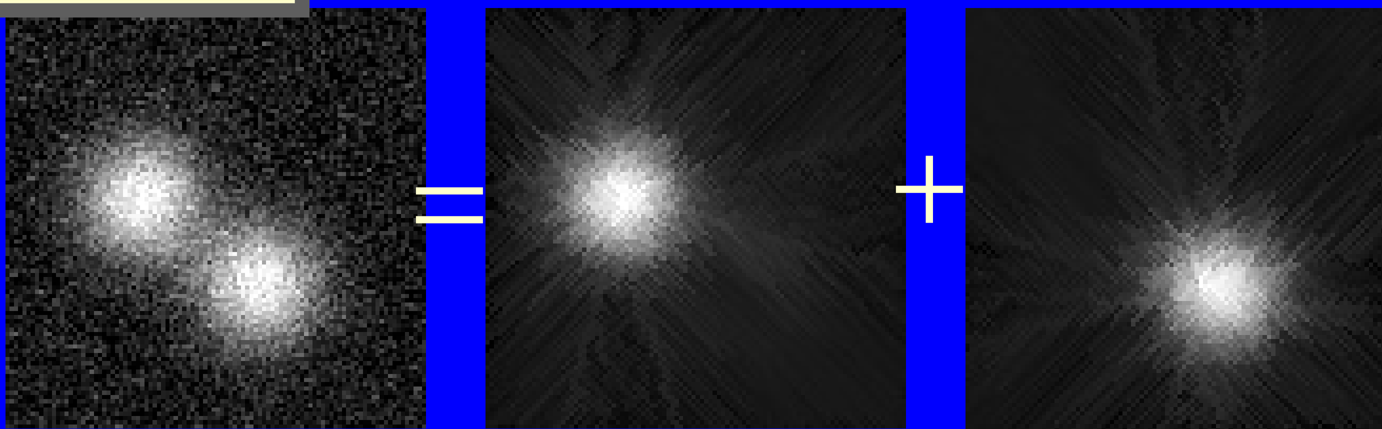
Need a slide here showing how to un-trap iteration in 1-D case.

Multiply fit for each galaxy by f, where f=1 at center and f<1 elsewhere. We use Gaussian. This sort of eats away at the shoulder. Results are nearly independent of f.

What you'll find is that f, and the rate at which you change it, controls how overlaps are allocated, more than anything else. Well separated galaxies are completely unaffected by f.

Two galaxies, convergence trap fixed.

Need more details here.

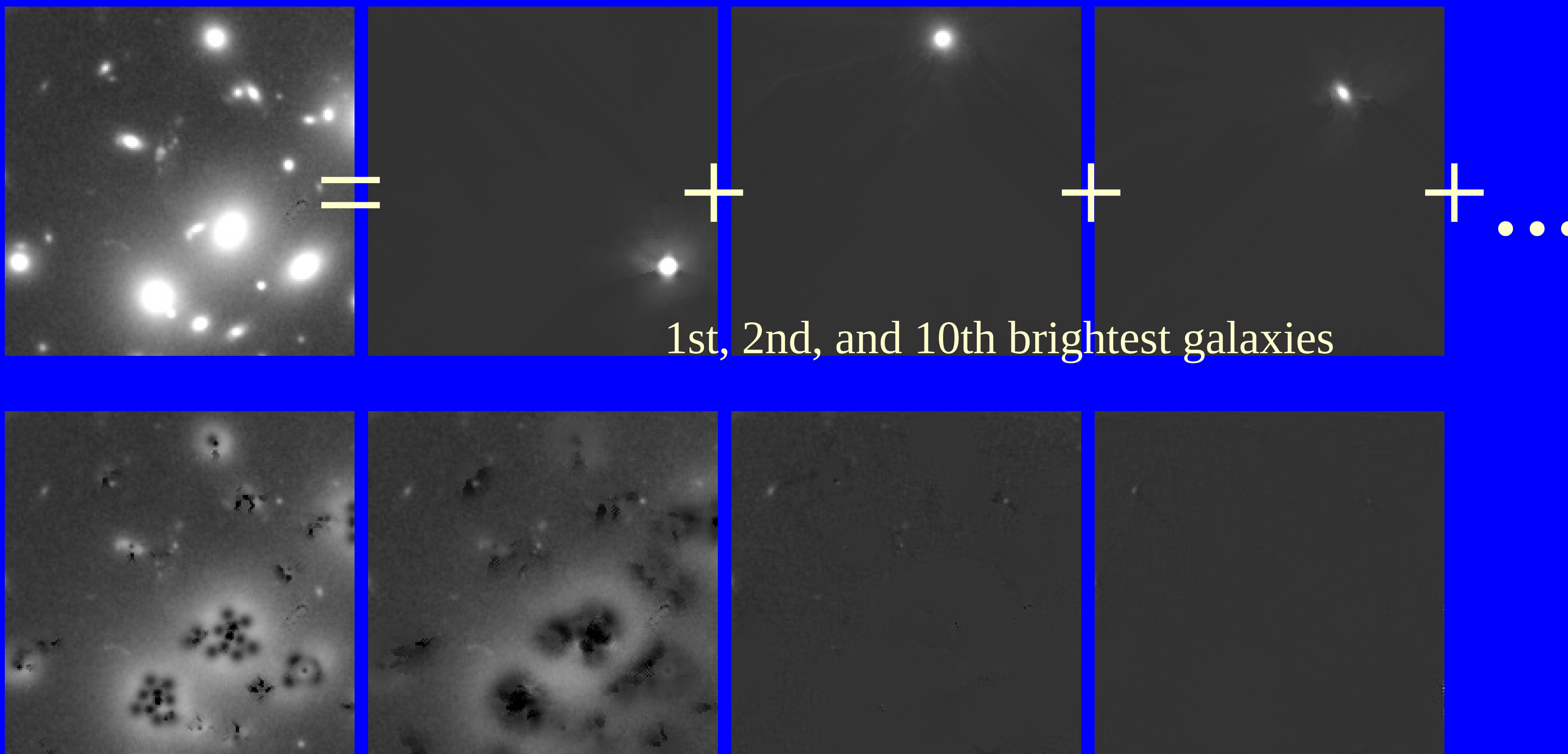


Each fit is multiplied by a masking Gaussian which starts small (3 pixels) so that each peak captures the light in its immediate neighborhood.

The mask is progressively broadened until it becomes flat and unity at the end of the run, so the residuals are driven to zero.

Final residual

HST data decomposition



1st, 2nd, and 10th brightest galaxies

Residuals at 1, 2, 4, 8 iterations

Plotted brightness = asinh(data)

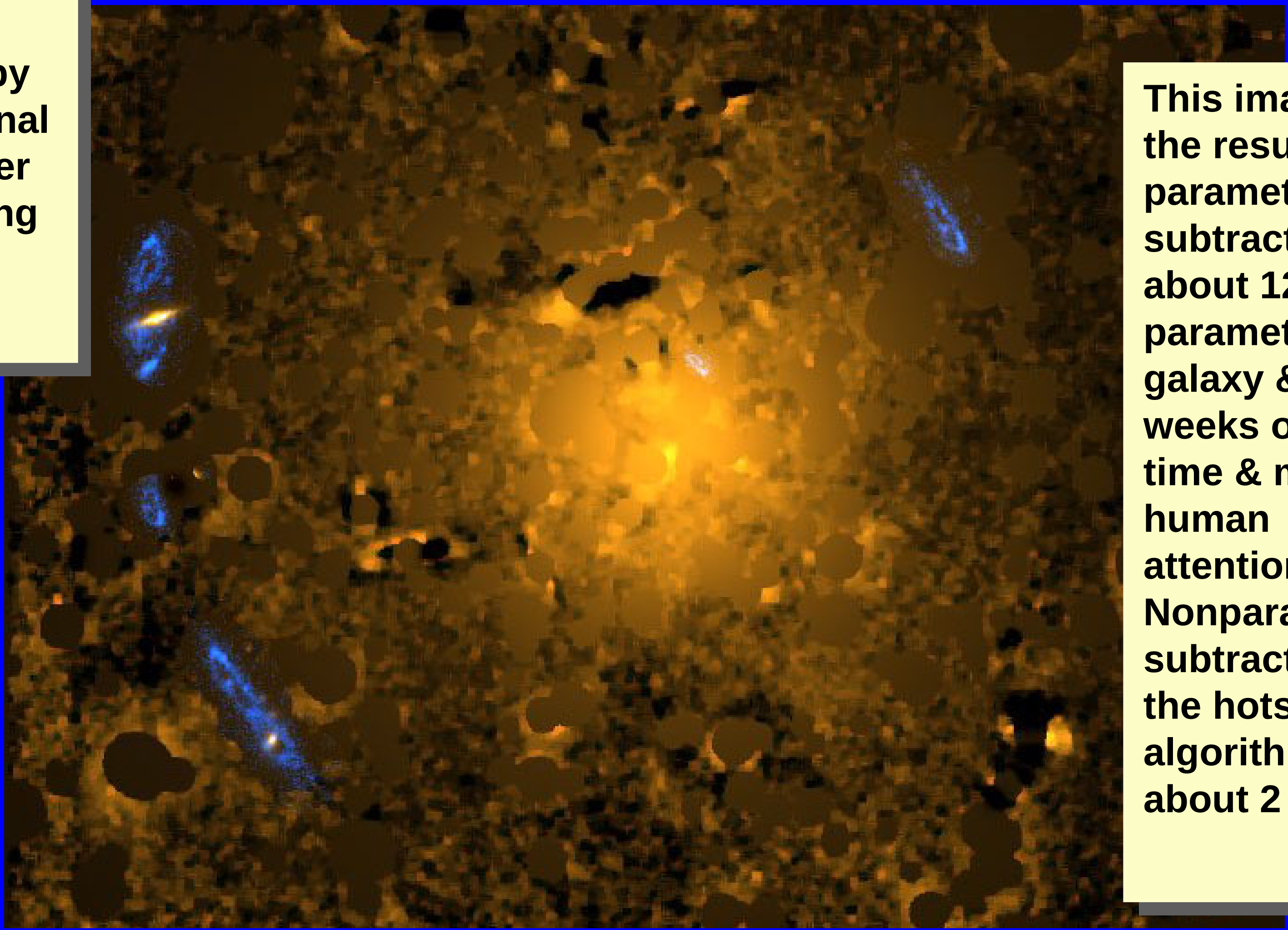
Science as a byproduct...

Hubble Space Telescope image of a cluster of galaxies (orange). The blue-white blobs are distorted images of a galaxy behind the cluster. The cluster is massive enough to bend space slightly, and let you see the same background galaxy around both sides of the cluster.

Those distorted images (called 'arcs' in the trade) are why we want to model and subtract galaxies.   We want to get rid of the cluster light.

Science as a byproduct...

Stars on the outskirts of a galaxy are loosely bound, and can be ripped away by the gravitational field of another galaxy, passing nearby.



This image is the result of a parametric subtraction, about 12 parameters per galaxy & 2 weeks of CPU time & much human attention. Nonparametric subtraction via the hotspot algorithm is about 2 hours.

Diffuse cluster light, from stars stripped out of galaxies by collisions between galaxies. Measurements of the diffuse light are important, as they help measure how big galaxies were as they cluster formed.

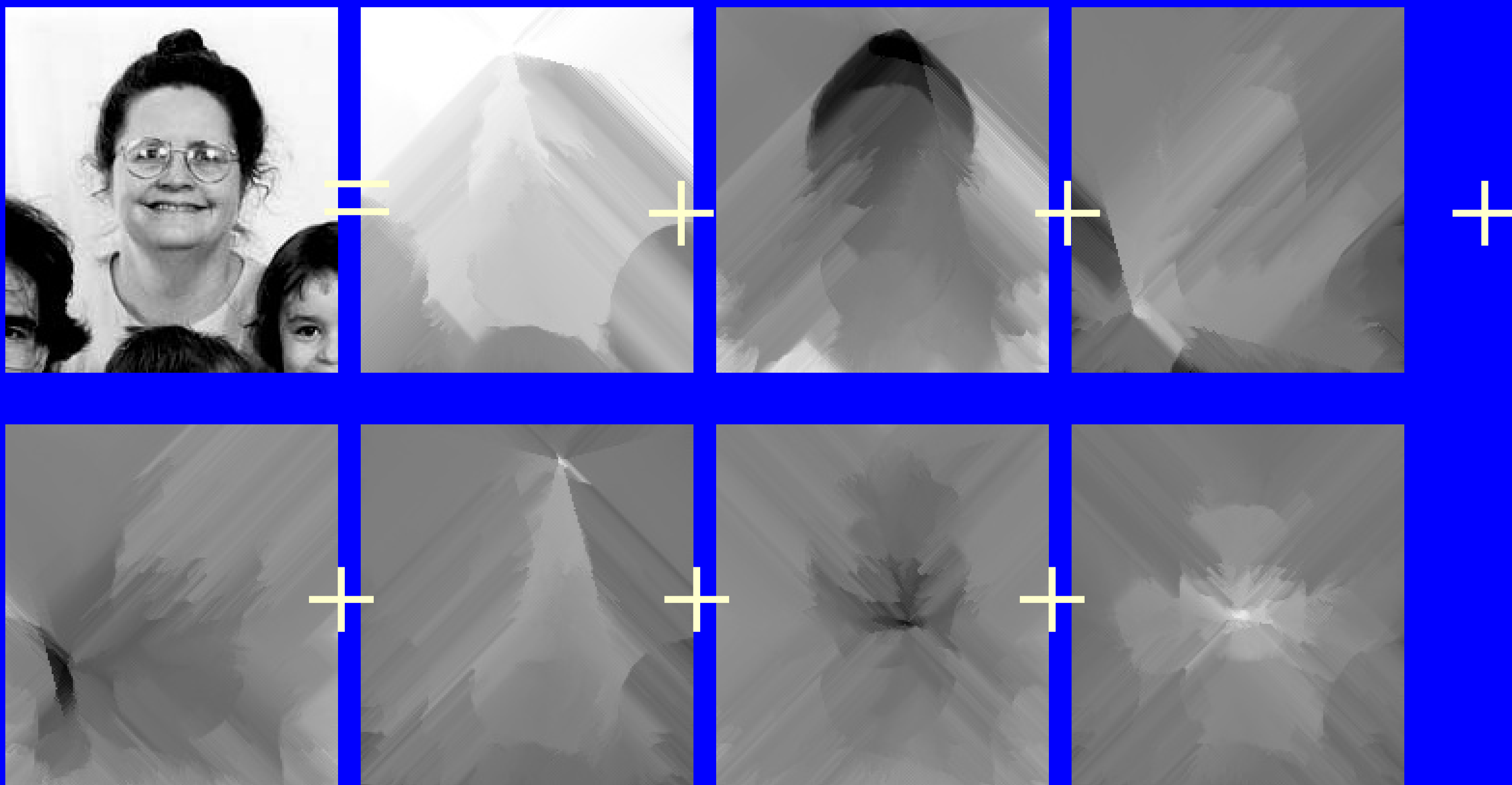
Science as a byproduct...

Density is measured by calculating how much mass needs to be in a given area to explain all the arcs (blue) as images of a single background galaxy.

Total cluster mass density (from gravitational lensing model).

The eternal question:

…but can you make money with it?



Seven terms, alternating positive and negative.

Data-driven image segmentation.

It's not immediately a compression algorithm: we compress one image into seven.

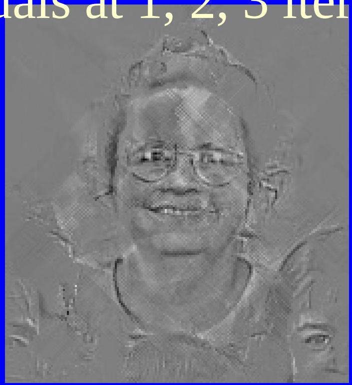
Each of the seven, though is lower entropy.

Segmenting faces...

Original

Reconstructed from 7 terms.

Residuals at 1, 2, 3 iterations



Conclusions

- The hotspot decomposition is a
  - nonparametric,
  - stable,
  - reliably convergent algorithm for taking apart images.
- It generates a set of monotonic components, at position driven by hotspots in the data.
- It will accurately reconstruct continuous tone images with remarkably few terms.